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Anomalous effective lagrangians and vector resonance models

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Chiral lagrangians including vector resonances have been shown to saturate the finite part of some of the counterterms needed to regularize ordinary one-loop effective lagrangians of pseudoscalar interactions with external currents. The equivalence between different models has been discussed in the ordinary case. In this paper we extend the analysis to the anomalous sector of chiral lagrangians by comparing the conventional vector model, the "hidden gauge symmetry" model, where vectors are gauge bosons of a spontaneously broken gauge symmetry, and the model where they are introduced through a tensor field. We find the equivalence works between the last two and that it can be recovered for the first only by including at least scalar resonances in the model. We modify the original formulations of these models by adding a "minimal coupling" hypothesis which reduces the number of couplings while preserving good agreement with experimental data. Within our scheme, in the absence of chiral breakings and of external axial currents, the anomalous lagrangian can be written in terms of a single known parameter and the saturation hypothesis can be seen to work in the photon-pseudoscalar sector.

1. Introduction

Effective chiral lagrangians describe the strong interactions of pseudoscalar mesons at low energy in the presence of external electromagnetic and weak currents. At leading order in an expansion in powers of external momenta p the effective action contains two sectors: (i) the non-anomalous sector, which is of order p^2 in the chiral expansion, and (ii) the anomalous sector, the Wess–Zumino action, which is of order p^4 and generates the chiral anomaly. The non-renormalizability of the theory implies that at next-to-leading order divergent contributions appear which are proportional to operators not present in the leading order lagrangian. One needs to add new counterterms and fix their finite part at a given energy through the comparison of chiral predictions for a given process with the experimental measurements. This analysis has been done in ref. [1] for the full set

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of counterterms of the next-to-leading lagrangian L_4 (order p^4 in the non-anomalous sector. However the procedure is difficult to implement beyond next-to-leading order, when the number of counterterms becomes very large.

A new approach has been proposed and developed by several authors in the last decade. It is based on the following observation: beyond leading order pseudoscalar meson interactions can be mediated by resonances (vector, axial–vector, scalar and pseudoscalar) which can give an effective contribution to the finite part of the counterterms at low energies. The value of this contribution can be expressed in terms of the resonance parameters.

The results of the analysis performed in ref. [2] for the L_4 counterterms show that most of the renormalized counterterms are essentially saturated by the resonance exchange at a scale $\mu = m_\rho$.

Three models have been proposed to describe the dynamics of vector resonances and their interactions with pseudoscalar mesons: (i) the Hidden Gauge Symmetry Model (HGS), formulated by Fujiwara et al. [3] and Bando et al. [4], in which the vector mesons are the gauge bosons of a hidden local symmetry $SU_V(3)$, (ii) the Conventional Vector model (CV) [5], in which the vector mesons are described as ordinary vector field and (iii) the Tensor model (T) with the vector mesons described as tensor fields [2].

Their equivalence or non-equivalence in predicting the low energy parameters of the pseudoscalar interactions is an open and interesting problem. Within the resonance saturation hypothesis their equivalence would lead to unambiguous predictions for some of the renormalized parameters of the low energy effective lagrangian. Ecker et al. [5] made a comparison of the three models in the non-anomalous sector at order p^4 of the effective lagrangian. In this case the Tensor and the HGS model turned out to be equivalent (but only for a particular choice of the values of the coefficients in the resonance lagrangian). The CV model requires the addition "by hand" of extra operators necessary to give the right prediction of the low energy observables.

In this paper we study the equivalence of vector resonance models in the next-to-leading $O(p^6)$ lagrangian of the anomalous sector L_6^{odd}, which is intrinsic parity odd. Contrary to the non-anomalous case, in this case a saturation hypothesis remains the only guide to make predictions, given the large number of counterterms which cannot be determined independently in terms of physical processes.

In sect. 2 we write explicitly the operatorial form of the one loop divergences of the Wess–Zumino action derived in refs. [6–8]. The complete $O(p^6)$ lagrangian (see ref. [7] for details) contains all the counterterms which absorb the one-loop divergences (called divergent terms from now on) and all the possible finite contributions which are compatible with the symmetry principle. Some of these finite terms will be produced by vector models. In sect. 3 we construct the most general lagrangians for the HGS, CV and T models which give contributions to the

$O(p^6)$ effective action (this work was done in part by Bando et al. [4] and Ecker et al. [5] for the HGS and CV models respectively). In the formulation of each model we make a new hypothesis, the "Minimal Coupling" hypothesis (MC), which improves their predictive power by reducing the number of coupling constants that we have to introduce. In sect. 4 we define a strong and a weak equivalence and prove the strong equivalence of the HGS and T models and the weak equivalence of both with the CV model. In sect. 5 some test processes are analysed to show the success of the "Minimal Coupling" hypothesis and fix with this assumption all the coupling constants of vector resonance models in the anomalous sector leading to the contributions of $O(p^6)$ in the effective theory.

Our conclusions are summarized in sect. 6. In appendix A we list some useful identities for the SU(3) octet operators and in appendix B the rules for the construction of the interaction vector lagrangian are worked out. In appendix C we construct the lowest order interaction lagrangian (with any intrinsic parity) of axial, scalar, pseudoscalar and singlet resonances and show how scalars can account for the operators of the HGS and T models which cannot be produced by the CV model.

2. Anomalous effective lagrangian at $O(p^6)$

The one-loop divergences of the Wess–Zumino action have been written by several authors (see refs. [6–8]). We write it in the form that we will use to compare the effective lagrangians derived from different vector resonance models:

$$\Delta L^{\infty}_{1\,\mathrm{loop}} = -\frac{1}{16\pi^2(d-4)}\left(\frac{N_c N_f}{72\pi^2 f^2}\epsilon^{\mu\nu\alpha\beta}\left\langle\left\langle\left[\frac{i}{4}\left[\mathscr{L}_\lambda,\, U^\dagger D^\lambda D^\mu U - D^\lambda D^\mu U^\dagger U\right]\right.\right.\right.\right.$$
$$-\frac{i}{4}\left[\mathscr{L}_\mu,\, U^\dagger D^2 U - D^2 U^\dagger U\right] - \tfrac{1}{2}\left[\mathscr{L}^\lambda,\, L_{\lambda\mu} - \tilde{R}_{\lambda\mu}\right]$$
$$\left. - U^\dagger D^\lambda R_{\lambda\mu} U - D^\lambda L_{\lambda\mu}\right]$$
$$\left.\left.\times\left[\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta + \frac{i}{2}\left(L_{\nu\alpha} + \tilde{R}_{\nu\alpha}\right)\mathscr{L}_\beta + \frac{i}{2}\mathscr{L}_\nu\left(L_{\alpha\beta} + \tilde{R}_{\alpha\beta}\right)\right]\right\rangle\right\rangle$$
$$-i\frac{N_c}{48\pi^2 f^2}\epsilon^{\mu\nu\alpha\beta}\left\langle\left\langle\left[\frac{N_f^2-4}{4N_f}\left(U^\dagger\chi + \chi^\dagger U\right) - \frac{N_f}{4}\mathscr{L}_\lambda\mathscr{L}^\lambda\right]\right.\right.$$
$$\times\left[\tilde{R}_{\mu\nu} L_{\alpha\beta} - L_{\mu\nu}\tilde{R}_{\alpha\beta} - i\left(\tilde{R}_{\mu\nu} - L_{\mu\nu}\right)\mathscr{L}_\alpha\mathscr{L}_\beta\right.$$

$$+i\mathscr{L}_\mu\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\Big]\Big\rangle - \frac{i}{2N_\mathrm{f}}\Big\langle\left(U^\dagger\chi+\chi^\dagger U\right)$$

$$\times\Big[\left(\tilde{R}_{\mu\nu}-L_{\mu\nu}\right)\mathscr{L}_\alpha\mathscr{L}_\beta-\mathscr{L}_\mu\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\Big]\Big\rangle$$

$$-\frac{i}{2}\left\langle\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\right\rangle\left\langle\mathscr{L}_\mu\left(U^\dagger\chi+\chi^\dagger U-\mathscr{L}_\lambda\mathscr{L}^\lambda\right)\right\rangle$$

$$-\frac{i}{2}\left\langle\mathscr{L}_\lambda\mathscr{L}_\mu\right\rangle\left\langle\mathscr{L}^\lambda\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)+\mathscr{L}^\lambda\left(\tilde{R}_{\nu\alpha}-L_{\nu\alpha}\right)\mathscr{L}_\beta\right\rangle\Bigg\}\Bigg]. \qquad (1)$$

$\langle\dots\rangle$ stands for the trace over the $SU(N_\mathrm{f})$ flavour indices and N_c is the number of QCD colours. For $N_\mathrm{f}=3$ the pseudoscalar field is described by the exponential representation $U=\exp(2i\Phi/f)$, where Φ is the 3×3 matrix of the pseudoscalar octet of SU(3) of flavour (π, K and η) and $f=132$ MeV is the pion decay constant. Under the group $\mathrm{G}=\mathrm{SU(3)_L}\times\mathrm{SU(3)_R}$ U transforms as

$$U \xrightarrow{\mathrm{G}} V_\mathrm{R} U V_\mathrm{L}^\dagger, \qquad (2)$$

and its covariant derivative is defined by

$$D_\mu U=\partial_\mu U-iR_\mu U+iUL_\mu, \qquad (3)$$

with the right and left-handed external currents R_μ and L_μ transforming as

$$R_\mu\to V_\mathrm{R}R_\mu V_\mathrm{R}^\dagger+iV_\mathrm{R}\partial_\mu V_\mathrm{R}^\dagger,$$

$$L_\mu\to V_\mathrm{L}L_\mu V_\mathrm{L}^\dagger+iV_\mathrm{L}\partial_\mu V_\mathrm{L}^\dagger. \qquad (4)$$

The "building blocks" in eq. (1) are defined as follows:

$$\mathscr{L}_\mu=U^\dagger D_\mu U,$$

$$L(R)_{\mu\nu}=\partial_\mu L(R)_\nu-\partial_\nu L(R)_\mu-i\left[L(R)_\mu,\,L(R)_\nu\right],$$

$$\tilde{R}_{\mu\nu}=U^\dagger R_{\mu\nu}U. \qquad (5)$$

The operator χ is the linear combination of scalar and pseudoscalar external fields via $\chi=2B_0(s+ip)$. The scalar field s is the quark mass matrix in a first approximation: $s=M+\dots$.

As Akhoury and Alfakih noticed in ref. [7], the $\Delta L^\infty_{1\,\mathrm{loop}}$ can be written in a more compact form, using a particular set of "building blocks". This set of operators is the same as the ones we will use to construct the resonance lagrangian for each model in turn. We define them here and describe their transformation properties in sect. 2. Introducing the square root of the U field $u = \exp(i\Phi/f)$, the operators are

$$u_\mu = iu^\dagger D_\mu U u^\dagger = u_\mu^\dagger,$$
$$f_{\mu\nu}^\pm = uL_{\mu\nu}u^\dagger \pm u^\dagger R_{\mu\nu}u,$$
$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u. \tag{6}$$

u_μ is an $O(p)$ operator and $f_{\mu\nu}^\pm$ and $\chi^\pm$ are $O(p^2)$ operators. All the one-loop divergences of the WZ action can be expressed in terms of u_μ, $f_{\mu\nu}^-$, $\chi^\pm$ and of a linear combination of these fields with their covariant derivatives:

$$\Upsilon_{\mu\nu} = f_{\mu\nu}^+ + \frac{i}{2}[u_\mu, u_\nu],$$
$$\nabla_\alpha \Upsilon_{\mu\nu} = uD_\alpha L_{\mu\nu}u^\dagger + u^\dagger D_\alpha R_{\mu\nu}u - \frac{i}{2}\left[u_\alpha, f_{\mu\nu}^-\right] + \frac{i}{2}\nabla_\alpha[u_\mu, u_\nu]. \tag{7}$$

This particular combination $\Upsilon_{\mu\nu}$ will play an essential role in the construction of the vector resonance models.

The compact form of $\Delta L_{1\,\text{loop}}^\infty$ reads

$$\begin{aligned}\Delta L_{1\,\text{loop}}^\infty = -\frac{1}{16\pi^2(d-4)}\frac{N_c}{72\pi^2 f^2}\epsilon^{\mu\nu\alpha\beta}\Bigg(& -\frac{N_f}{2}\left\langle \nabla^\lambda \Upsilon_{\lambda\mu}\{\Upsilon_{\nu\alpha}, u_\beta\}\right\rangle \\ & - i\frac{3(N_f^2-4)}{16N_f}\left\langle \chi_+\left[\Upsilon_{\mu\nu}, f_{\alpha\beta}^-\right]\right\rangle \\ & + \frac{3N_f}{16}\left\langle \chi_+\left[u_\mu u_\nu, f_{\alpha\beta}^-\right]\right\rangle \\ & - i\frac{3N_f}{16}\left\langle u_\lambda u^\lambda\left[\Upsilon_{\mu\nu}, f_{\alpha\beta}^-\right]\right\rangle \\ & + \frac{3N_f}{16}\left\langle u_\lambda u^\lambda\left[u_\mu u_\nu, f_{\alpha\beta}^-\right]\right\rangle \\ & - \frac{3}{4}\langle \chi_+ u_\mu\rangle\left\langle u_\nu f_{\alpha\beta}^-\right\rangle \\ & - \frac{3}{4}\left\langle u_\lambda u^\lambda u_\mu\right\rangle\left\langle u_\nu f_{\alpha\beta}^-\right\rangle \\ & + \frac{3}{4}\langle u_\lambda u_\mu\rangle\left\langle \{u_\nu, u^\lambda\} f_{\alpha\beta}^-\right\rangle\Bigg).\end{aligned} \tag{8}$$

In the latter construction the $O(p^6)$ lagrangian in the anomalous sector needs eight divergent counterterms. Alternative compact forms may need a different and lower number of counterterms. The maximum number of independent operators is 15.

3. Vector resonance models

The most general lagrangian in the anomalous sector is given by the set of all the independent, chiral invariant terms (mediating the interaction among vector mesons, pseudoscalar mesons and external gauge bosons) which are hermitian (i.e. a hermitian operator times a real coupling constant or an antihermitian operator times a purely imaginary coupling constant), invariant under P and C transformations and violate intrinsic parity.

In all models, we will use the set of operators defined in (6) as "building blocks" for the external currents (we uniformize to the language of refs. [2,5] for all the cases under study). Each one of these operators $\{\Theta\}$ involves the pseudoscalar field and the external gauge bosons fields. All the operators $\{\Theta\}$ transform as octets under G:

$$\Theta \xrightarrow{G} h(\Phi)\Theta h(\Phi)^{\dagger}, \tag{9}$$

where $h(\Phi)$ is the nonlinear realization of G which acts on the element $u(\Phi)$ of the coset space $SU(3)_L \times SU(3)_R / SU(3)_V$.

The spin-1 vector fields ($J^{PC} = 1^{--}$) and their transformation properties will be introduced separately for each model.

3.1. ONE- AND TWO-VECTOR INTERACTIONS

The lagrangian which describes the interactions of vector mesons with external currents can a priori include terms with an increasing number of vectors. We are interested in producing effective interactions of pseudoscalars at $O(p^6)$ in the anomalous sector of the effective theory, starting with the lagrangian of a vector resonance model. For this we need to construct external currents that interact with vector mesons up to a certain order in powers of momenta. Currents up to $O(p^4)$ are sufficient to our purposes. The interaction lagrangian can be written as a sum of one- and two-vector terms:

$$L_I = \langle VJ_1 \rangle + \tfrac{1}{2}\langle VVJ_2 \rangle. \tag{10}$$

The complete vector lagrangian includes the kinetic part:

$$L_V = \tfrac{1}{2}\langle V \Box V \rangle + \tfrac{1}{2}M^2\langle VV \rangle + \langle VJ_1 \rangle + \tfrac{1}{2}\langle VVJ_2 \rangle. \tag{11}$$

Terms with three vectors or more start to contribute at $O(p^8)$ in the anomalous sector. The currents J_1 and J_2 can be divided in an intrinsic parity even ("ordinary") and an intrinsic parity odd ("anomalous") part

$$J = J^{\mathrm{O}} + J^{\mathrm{A}}. \tag{12}$$

As we will see later in detail, the structure of the HGS and the T models (for the CV case this argument will not be necessary) requires that J_1^{O}, J_2^{O} and J_2^{A} start at $O(p^2)$, while J_1^{A} can start only at $O(p^4)$. Each term in L_{I} has total dimension $d = 4$ in momentum space and $d(V) = 1$. If one redefines each current as an object of $O(p^2)$ or $O(p^4)$ with dimensionless couplings and writes the appropriate dimensionful parameter (μ to some power) in front of each term, L_{I} reads

$$L_{\mathrm{I}} = \frac{1}{\mu}\left\langle VJ_1^{\mathrm{A}}(p^4)\right\rangle + \mu\left\langle VJ_1^{\mathrm{O}}(p^2)\right\rangle + \tfrac{1}{2}\left\langle VV\left(J_2^{\mathrm{A}}(p^2) + J_2^{\mathrm{O}}(p^2)\right)\right\rangle. \tag{13}$$

To find the effective lagrangian we have to integrate out the resonance field V (i.e. substitute the solution of the equation of motion in the resonance lagrangian). By minimizing the action

$$S_{\mathrm{V}} = \int \mathrm{d}^4x\, L_V \tag{14}$$

we obtain the classical equation of motion for the V field

$$\Box V + M^2 V + \left(J_2^{\mathrm{A}} + J_2^{\mathrm{O}}\right)V = -\left(\mu J_1^{\mathrm{O}} + \frac{1}{\mu}J_1^{\mathrm{A}}\right), \tag{15}$$

whose solution can be written as

$$V = -\left(M^2 + \left(J_2^{\mathrm{A}} + J_2^{\mathrm{O}}\right) + \Box\right)^{-1}\left(\mu J_1^{\mathrm{O}} + \frac{1}{\mu}J_1^{\mathrm{A}}\right). \tag{16}$$

Expanding the solution up to the order p^2 that we need in the "ordinary" part and up to $O(p^4)$ in the "anomalous" one, we get

$$V \simeq -\frac{\mu}{M^2}J_1^{\mathrm{O}}(p^2) - \frac{1}{M^2}\frac{1}{\mu}J_1^{\mathrm{A}}(p^4) + \frac{\mu}{\left(M^2\right)^2}J_1^{\mathrm{O}}(p^2)J_2^{\mathrm{A}}(p^2). \tag{17}$$

Terms at $O(p^4)$ "ordinary" and terms at $O(p^6)$ "anomalous" give contribution to the $O(p^6)$ "ordinary" and to the $O(p^8)$ "anomalous" terms respectively, in the effective lagrangian and are neglected. Inserting the solution (17) in the lagrangian (11) we obtain the effective lagrangian at $O(p^6)$, which is intrinsic parity odd:

$$
\begin{array}{ll}
L_V^{\text{odd}} = & L_{\text{eff}}^{\text{odd}} = \\[1ex]
\frac{1}{2}M^2\langle VV\rangle + \rightarrow & \frac{1}{M^2}\langle J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{A}}(p^4)\rangle \\[1ex]
 & -\frac{\mu^2}{M^4}\langle J_2^{\mathrm{A}}(p^2)J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{O}}(p^2)\rangle \\[1ex]
\frac{1}{\mu}\langle VJ_1^{\mathrm{A}}(p^4)\rangle + \rightarrow & -\frac{1}{M^2}\langle J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{A}}(p^4)\rangle \\[1ex]
\mu\langle VJ_1^{\mathrm{O}}(p^2)\rangle + \rightarrow & -\frac{1}{M^2}\langle J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{A}}(p^4)\rangle \\[1ex]
 & +\frac{\mu^2}{M^4}\langle J_2^{\mathrm{A}}(p^2)J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{O}}(p^2)\rangle \\[1ex]
\frac{1}{2}\langle VVJ_2^{\mathrm{A}}(p^2)\rangle \rightarrow & \frac{\mu^2}{2M^4}\langle J_2^{\mathrm{A}}(p^2)J_1^{\mathrm{O}}(p^2)J_1^{\mathrm{O}}(p^2)\rangle .
\end{array}
\tag{18}
$$

For the sum of the terms above we get

$$
L_{\text{eff}}^{\text{odd}} = -\frac{1}{M^2}\langle J_1^{\mathrm{O}}J_1^{\mathrm{A}}\rangle + \frac{\mu^2}{2M^4}\langle J_2^{\mathrm{A}}J_1^{\mathrm{O}}J_1^{\mathrm{O}}\rangle . \tag{19}
$$

If we compare the structure of one- and two-vector terms we see that the current $\tilde{J}_2^{\mathrm{A}} \equiv J_2^{\mathrm{A}}J_1^{\mathrm{O}}$ is a linear combination of operators already present in the J_1^{A} current, because the J_1^{A} is the most extended basis one can construct for the $O(p^4)$ "anomalous" currents which interact with the "ordinary" current J_1^{O}. Therefore the $\langle J_2^{\mathrm{A}}J_1^{\mathrm{O}}J_1^{\mathrm{O}}\rangle$ terms are a subset of the $\langle J_1^{\mathrm{A}}J_1^{\mathrm{O}}\rangle$ terms and the present formulation of the vector resonance model is somewhat redundant in predicting the effective interactions.

For this reason we formulate what we call a "Minimal Coupling" (MC) hypothesis. We define $\hat{J}_1^{\mathrm{A}}$ the set of operators of J_1^{A} which are not present in $\tilde{J}_2^{\mathrm{A}}$, i.e.

$$
J_1^{\mathrm{A}} = \tilde{J}_2^{\mathrm{A}} + \hat{J}_1^{\mathrm{A}}, \tag{20}
$$

and we restrict the J_1^A to its reduced form $\hat{J}_1^A$. Within the MC hypothesis, the effective lagrangian at $O(p^6)$ is given by

$$L_{\text{eff}}^{\text{odd}} = -\frac{1}{M^2}\langle J_1^O \hat{J}_1^A \rangle + \frac{\mu^2}{2M^4}\langle J_2^A J_1^O J_1^O \rangle, \tag{21}$$

with the vector field

$$V \simeq -\frac{\mu}{M^2} J_1^O(p^2).$$

If we choose the MC scheme we reduce the number of free coefficients in the resonance lagrangian. This procedure is safe only if a fit to all the experimental values of the resonances decay rates can be maintained at the level of the resonance lagrangian.

We will use the MC hypothesis in the construction of all the resonance models. In sect. 5 we will show that this choice is not excluded and even favoured by present experimental data.

3.2. THE HIDDEN GAUGE SYMMETRY MODEL

It was originally formulated in refs. [3,4]. This approach is based on the following theorem: any nonlinear σ-model symmetric under the coset group G/H is gauge equivalent to a linear model symmetric under the $G_{\text{global}} \times H_{\text{local}}$ group.

In our case vector mesons are introduced in the linear realization of a lagrangian symmetric under $(SU(3)_L \times SU(3)_R)_{\text{global}} \times SU(3)_{V_{\text{local}}}$ as the gauge bosons of the $SU(3)_V$ local symmetry (H_{local}) which is hidden in the usual nonlinear realization. The linear lagrangian contains an auxiliary field σ, represented by a unitary 3×3 matrix of unphysical scalar fields and transforming under the local group H as

$$\sigma \to U_H \sigma h^\dagger(\Phi). \tag{22}$$

It will be fixed equal to the unit matrix after unitary gauge fixing. The vector meson field is defined through the covariant derivative of σ as follows:

$$i\sigma^\dagger D_\mu \sigma \equiv i\sigma^\dagger \partial_\mu \sigma - g\sigma^\dagger \rho_\mu \sigma + v_\mu,$$

$$v_\mu \equiv \frac{1}{2i}\left[u^\dagger(\partial_\mu - iR_\mu)u + u(\partial_\mu - iL_\mu)u^\dagger\right],$$

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu + \partial_\nu \rho_\mu + ig[\rho_\mu, \rho_\nu]. \tag{23}$$

They transform under G as

$$\begin{aligned}\rho_\mu &\to U_{\rm H}\rho_\mu U_{\rm H}^\dagger - \frac{i}{g}U_{\rm H}\partial_\mu U_{\rm H}^\dagger,\\ v_\mu &\to h v_\mu h^\dagger - ih\partial_\mu h^\dagger,\\ i\sigma^\dagger D_\mu\sigma &\to ih\sigma^\dagger D_\mu\sigma h^\dagger,\\ \sigma^\dagger\rho_{\mu\nu}\sigma &\to h\sigma^\dagger\rho_{\mu\nu}\sigma h^\dagger. \end{aligned}\tag{24}$$

The last two terms transform as hermitian octets under G and we will use them as "building blocks" with one vector field in the construction of the lagrangian.

In the MC scheme, using the operators which transform under parity and charge conjugation as listed in table 1, the full lagrangian that gives a contribution at order p^6 in the anomalous sector of the effective theory is a sum of eight terms in the MC scheme

$$L = \sum_{i=1}^{8} a_i L_i,$$

which can be split into those terms which give contribution to the divergent part of $L_6^{\rm odd}$ (i.e. to the counterterms which absorb the one-loop divergences of the Wess–Zumino lagrangian)

Divergent contributions

$$\begin{aligned}&(1)\quad ig\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma\left\{u_\alpha,\sigma^\dagger D_\beta\sigma\right\}\right\rangle\\ &(2)\quad i\frac{g}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma\left[f_{\alpha\beta}^-, u_\lambda u^\lambda\right]\right\rangle,\\ &(3)\quad i\frac{g}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma\left[f_{\alpha\beta}^-,\chi_+\right]\right\rangle\end{aligned}\tag{25}$$

and into those terms which give only finite contributions to $L_6^{\rm odd}$

TABLE 1

P and C transformation properties of the building blocks of the HGS lagrangian. $\epsilon(\mu)$ is defined to be $\epsilon(0)=1$ and $\epsilon(i)=-1$. The σ field transforms as $\sigma^P=\sigma$ and $\sigma^C=\sigma^{\dagger T}$

	P	C
$i\sigma^\dagger D_\mu\sigma$	$\epsilon(\mu)$	$-(i\sigma^\dagger D_\mu\sigma)^{\rm T}$
$\sigma^\dagger\rho_{\mu\nu}\sigma$	$\epsilon(\mu)\epsilon(\nu)$	$-(\sigma^\dagger\rho_{\mu\nu}\sigma)^{\rm T}$
u_μ	$-\epsilon(\mu)$	$u_\mu^{\rm T}$
$f_{\mu\nu}^\pm$	$\pm\epsilon(\mu)\epsilon(\nu)$	$\mp f_{\mu\nu}^{\pm\,\rm T}$
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^{\rm T}$

Finite contributions

$$\begin{aligned}
&(1) \quad \frac{g^2}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma\left\{\nabla_\alpha\left(\sigma^\dagger\rho_{\beta\lambda}\sigma\right),\, u^\lambda\right\}\right\rangle,\\
&(2) \quad \frac{g^2}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma\left\{\sigma^\dagger\rho_{\alpha\beta}\sigma,\, \nabla_\lambda u^\lambda\right\}\right\rangle,\\
&(3) \quad i\frac{g}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\nu}\sigma u^\alpha \nabla^\lambda u_\lambda u^\beta\right\rangle,\\
&(4) \quad i\frac{g}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\left\langle\sigma^\dagger\rho_{\mu\lambda}\sigma u^\nu f_-^{\lambda\beta} u^\alpha\right\rangle,\\
&(5) \quad i\frac{g^2}{\mu^2}\epsilon_{\mu\nu\alpha\beta}\langle\chi_-\rangle\left\langle\left(\sigma^\dagger\rho_{\mu\nu}\sigma\right)\left(\sigma^\dagger\rho_{\alpha\beta}\sigma\right)\right\rangle,
\end{aligned} \tag{26}$$

where the covariant derivative $\nabla_\mu u_\nu$ is

$$\begin{aligned}
&\nabla_\mu u_\nu = \partial_\mu u_\nu - \left[\Gamma_\mu,\, u_\nu\right],\\
&\Gamma_\mu = i v_\mu,
\end{aligned} \tag{27}$$

with v_μ as in eq. (23). The terms (1) and (2) in the finite set contribute as independent reorderings of the Lorentz indices of the term (1) in the divergent set of the effective lagrangian. For example in the case of $\pi^0 \to \gamma\gamma$ decay they give corrections proportional to the pseudoscalar mass. Furthermore, the quantity $\nabla_\lambda u^\lambda$ in the finite set can be replaced through the classical equation of motion for the U field at $O(p^2)$ by

$$\nabla_\lambda u^\lambda = -\frac{i}{2}\chi_- + \frac{i}{6}\langle\chi_-\rangle. \tag{28}$$

If the chiral symmetry is explicitly broken the χ_+ terms and the terms (2) and (3) in the finite set give contributions proportional to the pseudoscalar masses in a first approximation, while the $\langle\chi_-\rangle$ term gives the suppressed contribution proportional to the SU(2) isospin breaking term $m_u - m_d$.

All the finite terms are zero in the chiral limit and without external non-chiral ($L \neq R$) gauge fields.

A few comments are in order at this point. If we do not choose the MC scheme we have to add two more independent terms in the divergent set. They are

$$\begin{aligned}
&(1a) \quad \epsilon_{\mu\nu\alpha\beta}\langle u_\mu u_\nu u_\alpha \sigma^\dagger D_\beta \sigma\rangle,\\
&(1b) \quad i\epsilon_{\mu\nu\alpha\beta}\left\langle f_{\mu\nu}^+\left\{u_\alpha,\, \sigma^\dagger D_\beta\sigma\right\}\right\rangle.
\end{aligned} \tag{29}$$

The term (1) gives a contribution to the effective lagrangian which is a particular linear combination of the contributions coming from terms (1a) and (1b). In the MC scheme we introduce only term (1), reducing the number of free coefficients as explained in subsect. 3.1. The same happens for the two-vector terms in the finite set. This approach differs from what has been done in previous works [6] where the full lagrangian was written as a sum of {1, 1a, 1b} divergent terms, without any MC hypothesis. In addition they did miss the terms (2) and (3) in the divergent set *.

3.3. THE CONVENTIONAL VECTOR MODEL

There is no compelling reason to assume the gauge boson nature of the spin-1 vector mesons. One can construct an alternative model to describe the dynamics and the interactions of these particles in which they are represented by an ordinary vector field V_μ. It transforms again as an octet under G, like in eq. (9), $V_\mu^P = \epsilon(\mu) V_\mu$ and $V_\mu^C = -V_\mu^T$ under parity and charge conjugation.

The kinetic lagrangian is

$$L_{\text{kin}} = -\tfrac{1}{4}\langle V_{\mu\nu} V^{\mu\nu}\rangle + \tfrac{1}{2} M^2 \langle V_\mu V^\mu \rangle, \tag{30}$$

with $V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$. The full lagrangian is obtained by adding the interaction part

$$L = L_{\text{kin}} + L_{\text{I}}. \tag{31}$$

The interaction lagrangian can start only at $O(p^3)$ for kinematical reasons. All the possible terms with one vector meson at lowest order, with any intrinsic parity are [5]

$$\begin{aligned} L_{\text{I}} &= \langle V_\mu J^\mu \rangle + \langle V_{\mu\nu} T^{\mu\nu} \rangle, \\ J_\mu &= i H_{\text{V}} [u^\nu, f_{\mu\nu}^-] + I_{\text{V}} [u_\mu, \chi_-] + i A_\epsilon \epsilon_{\mu\nu\alpha\beta} u^\nu u^\alpha u^\beta + B_\epsilon \epsilon_{\mu\nu\alpha\beta} \{u^\nu, f_+^{\alpha\beta}\}, \\ T_{\mu\nu} &= f_{\text{V}} f_{\mu\nu}^+ + i g_{\text{V}} [u_\mu, u_\nu], \end{aligned} \tag{32}$$

* This was due to the assumption that only terms with chiral dimensions fully matched by derivatives are the relevant ones [13]. By relaxing it one has to introduce new terms in the ordinary sector which, however, can be shown to give contributions already present through the anomalous part of the current.

leading to the following set of contributions:

$$
\begin{aligned}
&(1) \quad i\langle V^{\mu}[u^{\nu}, f^{-}_{\mu\nu}]\rangle,\\
&(2) \quad \langle V^{\mu}[u_{\mu}, \chi_{-}]\rangle,\\
&(3) \quad i\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu}u^{\nu}u^{\alpha}u^{\beta}\rangle,\\
&(4) \quad \epsilon_{\mu\nu\alpha\beta}\langle V^{\mu}\{u^{\nu}, f^{\alpha\beta}_{+}\}\rangle,\\
&(5) \quad \langle V^{\mu\nu}f^{+}_{\mu\nu}\rangle,\\
&(6) \quad i\langle V^{\mu\nu}[u_{\mu}, u_{\nu}]\rangle.
\end{aligned}
\tag{33}
$$

All these terms contribute to the divergent part of L_6^{odd} only. Finite contributions cannot be generated in this model and all terms which are the product of two traces in the flavour group are forbidden by P and C invariance.

New interaction terms with two vector mesons can be introduced also in this case, but they are not relevant. Indeed, the only kind of invariant interesting to our case in a "mass renormalization" term, $(1+\epsilon)M^2V_{\mu}V^{\mu}$ in the kinetic lagrangian. Because the ϵ correction starts at least at $O(p^2)$ there is no contribution coming from this correction at the order we are considering. There is no redundancy for the effective lagrangian at this order and the MC scheme is equivalent to the one already discussed in the literature.

3.4. THE TENSOR MODEL

In this model the vector mesons are represented by an antisymmetric tensor field $V_{\mu\nu}=-V_{\nu\mu}$ which possesses only six independent degrees of freedom. To have the three degrees of freedom appropriate to a massive vector field it is necessary to add further constraints to the most general kinetic lagrangian for a tensor field (see ref. [2] for more details). The kinetic lagrangian we use,

$$L_{\text{kin}} = -\tfrac{1}{2}\langle \nabla^{\lambda}V_{\lambda\mu}\nabla_{\nu}V^{\nu\mu}\rangle + \tfrac{1}{4}M^2\langle V_{\mu\nu}V^{\mu\nu}\rangle, \tag{34}$$

corresponds to the choice in which the V_{0i} components propagate while the V_{ij} degrees of freedom are frozen. The field $V_{\mu\nu}$ transforms as $V^{P}_{\mu\nu}=\epsilon(\mu)\epsilon(\nu)V_{\mu\nu}$, $V^{C}_{\mu\nu}=-V^{T}_{\mu\nu}$ under parity and charge conjugation. The group G acts on $V_{\mu\nu}$ with the usual nonlinear transformation for octet fields:

$$V_{\mu\nu} \xrightarrow{\text{G}} h(\Phi)V_{\mu\nu}h(\Phi)^{\dagger}. \tag{35}$$

The construction of the effective lagrangian for intrinsic parity violating processes at $O(p^6)$ requires the use of a set of relations that we describe in details in appendices A and B. There are no intrinsic parity odd terms with at least one vector meson at orders less than $O(p^4)$ for kinematical reasons (this guarantees the non-renormalization of the WZ action by vector meson exchange). The possible intrinsic parity odd interaction terms at lowest order, with one vector meson, have the following forms:

$$\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}\Omega^{\alpha\beta}(p^4)\rangle,$$

$$\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\lambda}\tilde{\Omega}_{\lambda}^{\nu\alpha\beta}(p^4)\rangle. \tag{36}$$

Two-vector terms are relevant and we use the MC hypothesis to construct all the interaction terms for the Tensor model. The possible contributions are:

Divergent contributions

$$\begin{aligned}
&(1)\quad \epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}\{\nabla_\lambda V^{\alpha\lambda}, u^\beta\}\rangle,\\
&(2)\quad \frac{i}{\mu}\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}[f_-^{\alpha\beta}, u^\lambda u_\lambda]\rangle,\\
&(3)\quad \frac{i}{\mu}\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}[f_-^{\alpha\beta}, \chi_+]\rangle.
\end{aligned}$$

Finite contributions

$$\begin{aligned}
&(1)\quad \epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}\{\nabla^\alpha V^{\beta\lambda}, u_\lambda\}\rangle,\\
&(2)\quad \epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}\{V^{\alpha\beta}, \nabla_\lambda u^\lambda\}\rangle,\\
&(3)\quad \frac{i}{\mu}\epsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}u^\alpha\nabla^\lambda u_\lambda u^\beta\rangle,\\
&(4)\quad \frac{i}{\mu}\epsilon_{\mu\nu\alpha\beta}\langle V_\lambda^\mu u^\nu f_-^{\lambda\beta}u^\alpha\rangle,\\
&(5)\quad i\epsilon_{\mu\nu\alpha\beta}\langle\chi_-\rangle\langle V^{\mu\nu}V^{\alpha\beta}\rangle.
\end{aligned} \tag{37}$$

The last term gives a finite contribution which is zero for SU(2) isospin exact $(m_u = m_d)$.

4. The equivalence of the models

We first define what we mean by equivalence of different resonance models. There are two degrees of equivalence: the one at the level of the resonances lagrangian and the one at the level of the effective lagrangian (after integrating out the resonance fields). The first implies the second, but not vice versa.

We are strictly interested in the second level of equivalence. Also in this case two types of equivalence can be distinguished. We will call them "strong equivalence" and "weak equivalence", with the following definitions.

Strong equivalence: the effective lagrangians in the two models have the same structure independently of the particular values of the coupling constants.

Weak equivalence: the two effective lagrangians can be made the same only by adding a set of ad hoc "local terms" of order p^6. These can belong to the set of divergent terms or to the one of finite terms. In the first case the model which needs the extra local terms is less predictive within a purely vector exchange scheme.

At the level of the effective lagrangian in the non-anomalous sector at $O(p^4)$ of the chiral expansion the HGS and T models are strongly equivalent, while the CV model satisfies only a weak equivalence with the first two [5].

We apply in detail to each model the procedure described in subsect. 3.1 which leads to the effective lagrangian.

4.1. $L_{\rm eff}$ FROM THE HGS MODEL

According to the MC scheme, the full lagrangian, including the intrinsic parity violating terms, can be written as follows:

$$\begin{aligned} L_{\rm HGS} = {} & f^2\left\langle \left(i\sigma^\dagger D_\mu \sigma\right)^2\right\rangle - \tfrac{1}{4}\left\langle \rho_{\mu\nu}^2\right\rangle \\ & + \frac{g}{\mu^2}\left\langle \sigma^\dagger \rho_{\mu\nu}\sigma J^{\mu\nu}_{\rm odd}(p^4)\right\rangle \\ & + \frac{g^2}{\mu^2}\left\langle \left(\sigma^\dagger \rho_{\mu\nu}\sigma\right)\left\{\sigma^\dagger\rho_{\alpha\beta}\sigma,\, J^{\mu\nu\alpha\beta}_{\rm odd}(p^2)\right\}\right\rangle \\ & + ig\left\langle \sigma^\dagger\rho_{\mu\nu}\sigma\left\{J^{\mu\nu\beta}_{\rm odd}(p),\, \sigma^\dagger D_\beta \sigma\right\}\right\rangle, \end{aligned} \tag{38}$$

where the explicit form of the external currents $J^{\mu\nu}_{\rm odd}(p^4)$, $J^{\mu\nu\alpha\beta}_{\rm odd}(p^2)$ and $J^{\mu\nu\beta}_{\rm odd}(p)$ can be deduced from eqs. (25) and (26) and the substitution $-iM^2\sigma^\dagger D_\mu\sigma = g\nabla^\lambda\rho^{\mu\lambda}$, valid at the effective lagrangian level, should be made. The solution of the equation of motion up to the order we need (p^3) is easily derived:

$$g\rho_\alpha = v_\alpha - (1/2f^2g^2)\nabla^\lambda v_{\lambda\alpha}, \tag{39}$$

where $v_{\lambda\alpha} \equiv \partial_\lambda v_\alpha - \partial_\alpha v_\lambda + i[v_\lambda, v_\alpha]$. From eq. (38) the relation $2f^2g^2 = M^2$ can be seen to hold.

We can write the effective lagrangian in the anomalous sector at $O(p^6)$ through the substitutions:

$$\begin{aligned}
-iM^2\sigma^\dagger D_\beta\sigma \to \nabla_\lambda v^{\beta\lambda} = {}& \frac{i}{8}u^\dagger\big(D_\lambda U D^\lambda U^\dagger D_\beta U + D_\beta U D_\lambda U^\dagger D^\lambda U \\
& - 2D_\lambda U D_\beta U^\dagger D^\lambda U\big)u^\dagger - \frac{i}{4}u\big(D_\lambda D_\beta U^\dagger D^\lambda U \\
& - D^\lambda U^\dagger D_\lambda D_\beta U \\
& + D_\beta U^\dagger D^2 U - D^2 U^\dagger D_\beta U\big)u^\dagger \\
& + \tfrac{1}{4}u\big[\mathscr{L}_\lambda, \tilde{R}_{\beta\lambda}\big]u^\dagger - \tfrac{1}{4}u\big[\mathscr{L}_\lambda, L_{\beta\lambda}\big]u^\dagger \\
& - \tfrac{1}{2}u^\dagger D^\lambda R_{\beta\lambda}u - \tfrac{1}{2}uD^\lambda L_{\beta\lambda}u^\dagger, \\
g\rho_{\mu\nu} \to v_{\mu\nu} = {}& -\frac{i}{4}uD_\mu U^\dagger D_\nu U u^\dagger + \frac{i}{4}uD_\nu U^\dagger D_\mu U u^\dagger \\
& - \tfrac{1}{2}u^\dagger R_{\mu\nu}u - \tfrac{1}{2}uL_{\mu\nu}u^\dagger.
\end{aligned} \tag{40}$$

Only the divergent terms are explicitly written:

$$\begin{aligned}
L_{\text{eff}}^{\text{odd}}(p^6) = \epsilon_{\mu\nu\alpha\beta}\Bigg\langle\Bigg\{ & \frac{i}{8}\big[\mathscr{L}_\lambda, U^\dagger D^\lambda D^\mu U - D^\lambda D^\mu U^\dagger U\big] \\
& - \frac{i}{8}\big[\mathscr{L}_\mu, U^\dagger D^2 U - D^2 U^\dagger U\big] - \tfrac{1}{4}\big[\mathscr{L}^\lambda, L_{\lambda\mu} - \tilde{R}_{\lambda\mu}\big] \\
& - \tfrac{1}{2}U^\dagger D^\lambda R_{\lambda\mu}U - \tfrac{1}{2}D^\lambda L_{\lambda\mu}\Bigg\}\Bigg\{-\frac{h_1}{2M^2}\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta \\
& - i\frac{h_1}{4M^2}\Big[\big(L_{\nu\alpha} + \tilde{R}_{\nu\alpha}\big)\mathscr{L}_\beta + \mathscr{L}_\nu\big(L_{\alpha\beta} + \tilde{R}_{\alpha\beta}\big)\Big]\Bigg\} \\
& + i\big\{h_3\mathscr{L}_\lambda\mathscr{L}^\lambda - h_2(U^\dagger\chi + \chi^\dagger U)\big\} \\
& \times\Bigg\{\tilde{R}_{\mu\nu}L_{\alpha\beta} - L_{\mu\nu}\tilde{R}_{\alpha\beta} - \frac{i}{2}\big(\tilde{R}_{\mu\nu} - L_{\mu\nu}\big)\mathscr{L}_\alpha\mathscr{L}_\beta
\end{aligned}$$

$$+\frac{i}{2}\mathscr{L}_{\mu}\mathscr{L}_{\nu}\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\Big\rangle\Big\rangle, \tag{41}$$

with h_2 and h_3 dimensionful parameters and h_1 dimensionless. Comparing with eq. (1) we realize that the 1^{--} vector mesons exchange can give contribution only to a part of the possible counterterms in the anomalous sector, as in the non-anomalous case. This is what we expected, knowing that the complete saturation of all the counterterms can come only from the all resonance exchange (vector, axial-vector, pseudoscalar and scalar).

The last two terms with coefficients h_2 and h_3 were not present in the previous formulations of the model [6], as we noticed before.

If we relax the MC hypothesis new terms are added both in the ordinary and the anomalous sector. By defining the ordinary current of order p^2 as

$$J_{\mathrm{O}}^{\mu\nu}=ic_1[u_\mu, u_\nu]+c_2 f_{\mu\nu}^{+}, \tag{42}$$

i.e. as an arbitrary linear combination of the operators already present in the $v_{\mu\nu}$ term of eq. (40), the full lagrangian is modified as follows:

$$\begin{aligned}L_{\mathrm{HGS}}=&f^2\Big\langle\left(i\sigma^{\dagger}D_{\mu}\sigma\right)^2\Big\rangle-\tfrac{1}{4}\langle\rho_{\mu\nu}^2\rangle+\langle\sigma^{\dagger}\rho_{\mu\nu}\sigma J_{\mathrm{O}}^{\mu\nu}(p^2)\rangle\\&+\frac{g}{\mu^2}\langle\sigma^{\dagger}\rho_{\mu\nu}\sigma J_{\mathrm{odd}}^{\mu\nu}(p^4)\rangle\\&+\frac{g^2}{\mu^2}\Big\langle\left(\sigma^{\dagger}\rho_{\mu\nu}\sigma\right)\left\{\sigma^{\dagger}\rho_{\alpha\beta}\sigma,\, J_{\mathrm{odd}}^{\mu\nu\alpha\beta}(p^2)\right\}\Big\rangle\\&+ig\Big\langle\sigma^{\dagger}\rho_{\mu\nu}\sigma\left\{J_{\mathrm{odd}}^{\mu\nu\beta}(p),\,\sigma^{\dagger}D_{\beta}\sigma\right\}\Big\rangle+(1\mathrm{a})+(1\mathrm{b}),\end{aligned} \tag{43}$$

where the terms (1a), (1b) are those defined in eq. (29). The solution (39) of the equation of motion becomes

$$g\rho_{\alpha}=v_{\alpha}-(1/2f^2g^2)\nabla^{\lambda}v_{\lambda\alpha}+(1/f^2g^2)\nabla^{\lambda}J_{\lambda\alpha}^{\mathrm{o}} \tag{44}$$

and the substitution for $-iM^2\sigma^{\dagger}D_{\beta}\sigma$ used to obtain the effective lagrangian is

$$-iM^2\sigma^{\dagger}D_{\beta}\sigma\to\nabla_{\lambda}v^{\beta\lambda}-2g\nabla_{\lambda}J_{\mathrm{O}}^{\beta\lambda}. \tag{45}$$

The above construction contains the maximum number of free parameters in the anomalous effective lagrangian at order p^6. Only the term with coefficient h_1, in eq. (41) gets modified: each one of the four blocks

$$\begin{aligned}
&(1)\quad i\left[\mathscr{L}_\lambda, U^\dagger D^\lambda D^\mu U - D^\lambda D^\mu U^\dagger U\right] - i\left[\mathscr{L}_\mu, U^\dagger D^2 U - D^2 U^\dagger U\right],\\
&(2)\quad \tfrac{1}{2}\left[\mathscr{L}^\lambda, L_{\lambda\mu} - \tilde{R}_{\lambda\mu}\right] + U^\dagger D^\lambda R_{\lambda\mu} U + D^\lambda L_{\lambda\mu},\\
&(3)\quad \left(L_{\nu\alpha} + \tilde{R}_{\nu\alpha}\right)\mathscr{L}_\beta + \mathscr{L}_\nu\left(L_{\alpha\beta} + \tilde{R}_{\alpha\beta}\right),\\
&(4)\quad \mathscr{L}_\nu \mathscr{L}_\alpha \mathscr{L}_\beta,
\end{aligned} \tag{46}$$

is now multiplied by a linear combination of the free parameters allowed. The ordinary interaction term $\langle \sigma^\dagger \rho_{\mu\nu} \sigma J_{\mathrm{O}}^{\mu\nu}(p^2)\rangle$ added in the general scheme also modifies the ordinary effective lagrangian at order p^4. The result of this arbitrariness permits the full equivalence of the HGS and Tensorial models without imposing the constraint $F_{\mathrm{V}} = 2G_{\mathrm{V}}$ on the parameters of the T model. This relation, like all the others imposed by the MC hypothesis, are phenomenologically well verified.

4.2. L_{eff} FROM THE CV MODEL AND ITS EQUIVALENCE WITH THE HGS MODEL

The effective lagrangian for the vector model, following eqs. (30) and (32), is:

$$L_{\mathrm{V}}^{\mathrm{eff}} = \tfrac{1}{2}\left\{\left\langle V_\mu J^\mu \right\rangle + \left\langle V_{\mu\nu} T^{\mu\nu}\right\rangle\right\}, \tag{47}$$

where the vector field V_μ is the solution of the equation of motion

$$\nabla_\lambda V^{\lambda\mu} + M_{\mathrm{V}}^2 V^\mu = -J^\mu - \nabla_\lambda (T^{\mu\lambda} - T^{\lambda\mu}) \tag{48}$$

and the external currents J^μ and $T^{\mu\nu}$ were defined in (32). Explicitly substituting the solution of the equation of motion, the effective lagrangian reads:

$$\begin{aligned}
L_{\mathrm{V}}^{\mathrm{eff}}(p^6) &= -\frac{1}{2M^2}\left\langle J_\mu J^\mu \right\rangle - \frac{1}{2M^2}\left\langle \nabla_\nu (T^{\mu\nu} - T^{\nu\mu}) J_\mu \right\rangle\\
&\quad - \frac{1}{2M^2}\left\langle (\nabla_\mu J_\nu - \nabla_\nu J_\mu) T^{\mu\nu}\right\rangle + \mathrm{O}(p^8)\\
&= -\frac{1}{2M^2}\left\langle J_\mu J^\mu \right\rangle - \frac{1}{M^2}\left\langle (\nabla_\mu J_\nu - \nabla_\nu J_\nu) T^{\mu\nu}\right\rangle\\
&\quad - \frac{1}{2M^2}\left\langle \nabla_\lambda (T^{\mu\lambda} - T^{\lambda\mu}) \nabla_\nu (T^{\mu\nu} - T^{\nu\mu})\right\rangle + \mathrm{O}(p^8).
\end{aligned} \tag{49}$$

The r.h.s. of eq. (49) contains an ordinary part (i.e. intrinsic parity even) and a part with the ϵ tensor (i.e. intrinsic parity odd) that we want to analyse. Writing explicitly the building blocks in terms of the pseudoscalar field and the gauge bosons fields, we obtain the effective lagrangian at $O(p^6)$

$$
\begin{aligned}
L_{\mathrm{eff}}^{\mathrm{odd}}(p^6) = -\frac{1}{2M^2}\epsilon_{\mu\nu\alpha\beta}\Big\langle\Big\{&\left[\mathscr{L}_\lambda, U^\dagger D^\lambda D^\mu U - D^\lambda D^\mu U^\dagger U\right] \\
&-\left[\mathscr{L}_\mu, U^\dagger D^2 U - D^2 U^\dagger U\right]\Big\} \\
&\times\left(-2g_{\mathrm{V}}\boldsymbol{B}_\epsilon\left\{\mathscr{L}_\nu, L_{\alpha\beta}+\tilde{R}_{\alpha\beta}\right\} + 2ig_{\mathrm{V}}\boldsymbol{A}_\epsilon\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right) \\
&-\left(U^\dagger D^\lambda R_{\lambda\mu}U + D^\lambda L_{\lambda\mu}\right) \\
&\times\left(4if_{\mathrm{V}}\boldsymbol{B}_\epsilon\left\{\mathscr{L}_\nu, L_{\alpha\beta}+\tilde{R}_{\alpha\beta}\right\} + 4f_{\mathrm{V}}\boldsymbol{A}_\epsilon\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right) \\
&+\left[\mathscr{L}^\lambda, L_{\lambda\mu}-\tilde{R}_{\lambda\mu}\right] \\
&\times 2\left(i(\boldsymbol{H}_{\mathrm{V}}-f_{\mathrm{V}})\boldsymbol{B}_\epsilon\left\{\mathscr{L}_\nu, L_{\alpha\beta}+\tilde{R}_{\alpha\beta}\right\} + (\boldsymbol{H}_{\mathrm{V}}-f_{\mathrm{V}})\boldsymbol{A}_\epsilon\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right) \\
&+2\left[\mathscr{L}_\mu, \chi^\dagger U - U^\dagger\chi\right]\left(\boldsymbol{I}_{\mathrm{V}}\boldsymbol{B}_\epsilon\left\{\mathscr{L}_\nu, L_{\alpha\beta}+\tilde{R}_{\alpha\beta}\right\} - i\boldsymbol{I}_{\mathrm{V}}\boldsymbol{A}_\epsilon\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right)\Big\rangle, \qquad (50)
\end{aligned}
$$

where the last term can be modified through a relation due to the equation of motion (28) for the pseudoscalar field

$$
\left[\mathscr{L}_\mu, \chi^\dagger U - U^\dagger\chi\right] = \left[\mathscr{L}_\mu, U^\dagger D^2 U - D^2 U^\dagger U\right]. \qquad (51)
$$

By comparing the two lagrangians (41) and (50) we can analyse the equivalence of the HGS and CV models. Our comparison is limited to the divergent terms because all finite terms cannot be produced by the Conventional Vector model. The "strong" equivalence is manifestly not verified, because of the presence of two more terms (the last ones in (41)) in the HGS model. All the couplings of the CV model are fixed in terms of the HGS coupling constants g and h_1. The independent relations we find are

$$
\text{(I)} \quad H_{\mathrm{V}} = I_{\mathrm{V}} = 0,
$$

$$
\text{(II)} \quad f_{\mathrm{V}} = \frac{1}{4g},
$$

$$
\text{(III)} \quad f_{\mathrm{V}} = 2g_{\mathrm{V}},
$$

$$\text{(IV)} \quad A_\epsilon = 2B_\epsilon,$$

$$\text{(V)} \quad f_V A_\epsilon = \frac{h_1}{8}. \tag{52}$$

The couplings h_2 and h_3, which are present in the HGS model and absent in the CV model, must be supplied explicitly to the second one as "local terms". However the addition of extra axial, pseudoscalar, scalar and flavour singlet resonances may generate the missing terms of the model, as well as the finite parts of all the counterterms of the one-loop effective action which cannot be obtained from any vector model.

There are four terms in the effective lagrangian that are not produced in the CV model with only vector mesons:

$$\begin{aligned}
&(1) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle [u_\mu, u_\nu][f^-_{\alpha\beta}, u_\lambda u^\lambda]\right\rangle,\\
&(2) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle [u_\mu, u_\nu][f^-_{\alpha\beta}, \chi_+]\right\rangle,\\
&(3) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle f^+_{\mu\nu}[f^-_{\alpha\beta}, u_\lambda u^\lambda]\right\rangle,\\
&(4) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle f^+_{\mu\nu}[f^-_{\alpha\beta}, \chi^+]\right\rangle.
\end{aligned} \tag{53}$$

As we show in detail in appendix C all these terms are generated by a scalar resonance exchange, while the axial resonance exchange can produce the terms (1) and (2) (plus a set of "finite" contributions). By considering only vector resonances, we can conclude that the CV model is less predictive than the HGS.

Notice, however, that the four operators above appear in the one loop effective lagrangian not only in the combination that the HGS model would require.

4.3. $L_{\rm eff}$ FROM THE T MODEL AND ITS EQUIVALENCE WITH THE HGS AND CV MODELS

We write the full lagrangian with external currents of orders p^2 and p^4 for the tensor field as

$$\begin{aligned}
L = &-\tfrac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu} \nabla_\nu V^{\nu\mu}\right\rangle + \tfrac{1}{4}M^2\left\langle V_{\mu\nu}V^{\mu\nu}\right\rangle\\
&+ \langle VJ_1\rangle + \langle VVJ_2\rangle.
\end{aligned} \tag{54}$$

Adopting the MC scheme, the currents J_1 and J_2 are defined as the sum of an "ordinary" part and an "anomalous" part following the results of subsect. 3.1:

$$\begin{aligned}
J_1 &\equiv \mu J_1^{\rm O}(p^2) + \frac{1}{\mu}\hat{J}_1^{\rm A}(p^4)\\
J_2 &\equiv J_2^{\rm O}(p^2) + J_2^{\rm A}(p^2).
\end{aligned} \tag{55}$$

Following again subsect. 3.1 we know that the effective lagrangian is given by

$$L_{\mathrm{V}}^{\mathrm{odd}} = \frac{1}{\mu}\langle V\hat{J}_1^{\mathrm{A}}\rangle + \langle VVJ_2^{\mathrm{A}}\rangle, \tag{56}$$

through the substitution

$$V = -2\mu/M^2 J_1^{\mathrm{O}}. \tag{57}$$

If we take into account the explicit form of the divergent contributions, contained in the list (37), we can write the $L_{\mathrm{V}}^{\mathrm{odd}}$ lagrangian as

$$L_{\mathrm{V}}^{\mathrm{odd}} = \epsilon^{\mu\nu\alpha\beta}\Big(t_1\big\langle V_{\mu\nu}\{u_\alpha, \nabla^\lambda V_{\beta\lambda}\}\big\rangle + it_2\big\langle V_{\mu\nu}\big[f_{\alpha\beta}^-, \chi_+\big]\big\rangle$$
$$+ it_3\big\langle V_{\mu\nu}\big[f_{\alpha\beta}^-, u_\lambda u^\lambda\big]\big\rangle\Big). \tag{58}$$

Comparing with eq. (56) we obtain the explicit form of the $\hat{J}_1^{\mathrm{A}}$ and J_2^{A} currents:

$$\frac{1}{\mu}\hat{J}_{1\mu\nu}^{\mathrm{A}} \equiv \epsilon_{\mu\nu\alpha\beta}\big\{it_2\big[f_-^{\alpha\beta}, \chi_+\big] > + it_3\big[f_-^{\alpha\beta}, u_\lambda u^\lambda\big]\big\},$$

$$J_{2\mu\nu\rho\sigma}^{\mathrm{A}} \equiv t_1\Big\{-\epsilon_{\rho\sigma\alpha\mu}\overleftarrow{\nabla}_\nu u^\alpha - \epsilon_{\rho\sigma\alpha\mu}\overrightarrow{\nabla}_\nu u^\alpha + \epsilon_{\mu\nu\alpha\rho}\overleftarrow{\nabla}_\sigma u^\alpha\Big\}. \tag{59}$$

We define J_1^{O} with couplings F_{V} and G_{V} already used in refs. [5,2]

$$\mu J_{1\mu\nu}^{\mathrm{O}} \equiv \frac{F_{\mathrm{V}}}{2\sqrt{2}} f_{\mu\nu}^+ + i\frac{G_{\mathrm{V}}}{2\sqrt{2}}[u_\mu, u_\nu]. \tag{60}$$

The effective lagrangian is

$$L_{\mathrm{eff}}^{\mathrm{odd}}(p^6) = \epsilon_{\mu\nu\alpha\beta}\Big\langle t_1\Big(U^\dagger D^\lambda R_{\lambda\mu}U + D^\lambda L_{\lambda\mu} + \tfrac{1}{2}\big[\mathscr{L}^\lambda, L_{\lambda\mu} - \tilde{R}_{\lambda\mu}\big]\Big)$$
$$\times\left(i\left(\frac{F_{\mathrm{V}}}{\sqrt{2}M^2}\right)^2\big\{L_{\nu\alpha} + \tilde{R}_{\nu\alpha}, \mathscr{L}_\beta\big\} + \frac{2F_{\mathrm{V}}G_{\mathrm{V}}}{M^4}\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right)$$
$$+t_1\Big(\big[\mathscr{L}_\lambda, U^\dagger D^\lambda D^\mu U - D^\lambda D^\mu U^\dagger U\big] - \big[\mathscr{L}_\mu, U^\dagger D^2 U - D^2 U^\dagger U\big]\Big)$$
$$\times\left(\frac{F_{\mathrm{V}}G_{\mathrm{V}}}{4M^4}\big\{L_{\nu\alpha} + \tilde{R}_{\nu\alpha}, \mathscr{L}_\beta\big\} - 2i\left(\frac{G_{\mathrm{V}}}{\sqrt{2}M^2}\right)^2\mathscr{L}_\nu\mathscr{L}_\alpha\mathscr{L}_\beta\right)$$

$$+i(U^\dagger\chi+\chi^\dagger U)\left(-2\frac{t_2F_\mathrm{V}}{\sqrt{2}M^2}\left(\tilde{R}_{\mu\nu}L_{\alpha\beta}-L_{\mu\nu}\tilde{R}_{\alpha\beta}\right)\right.$$

$$\left.+2i\frac{t_2G_\mathrm{V}}{\sqrt{2}M^2}\left[\left(\tilde{R}_{\mu\nu}-L_{\mu\nu}\right)\mathscr{L}_\alpha\mathscr{L}_\beta-\mathscr{L}_\mu\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\right]\right)$$

$$-i\mathscr{L}_\lambda\mathscr{L}^\lambda\left(-2\frac{t_3F_\mathrm{V}}{\sqrt{2}M^2}\left(\tilde{R}_{\mu\nu}L_{\alpha\beta}-L_{\mu\nu}\tilde{R}_{\alpha\beta}\right)\right.$$

$$\left.\left.+2i\frac{t_3G_\mathrm{V}}{\sqrt{2}M^2}\left[\left(\tilde{R}_{\mu\nu}-L_{\mu\nu}\right)\mathscr{L}_\alpha\mathscr{L}_\beta-\mathscr{L}_\mu\mathscr{L}_\nu\left(\tilde{R}_{\alpha\beta}-L_{\alpha\beta}\right)\right]\right)\right\rangle. \tag{61}$$

Comparing with eq. (41) we conclude that the HGS and the T models generate the same divergent contributions to the effective lagrangian at $O(p^6)$. All the couplings in the divergent set of the Tensor model are then related to the couplings of the divergent set of the HGS model by the following relations:

$$\begin{aligned}
&(\mathrm{I}) \quad 4F_\mathrm{V}^2t_1=M^2h_1,\\
&(\mathrm{II}) \quad \frac{F_\mathrm{V}}{M}=\frac{1}{\sqrt{2}g}\xRightarrow{M=\sqrt{2}fg}F_\mathrm{V}=f,\\
&(\mathrm{III}) \quad F_\mathrm{V}=2G_\mathrm{V},\\
&(\mathrm{IV}) \quad t_2=\frac{M^2}{\sqrt{2}F_\mathrm{V}}h_2,\\
&(\mathrm{V}) \quad t_3=\frac{M^2}{\sqrt{2}F_\mathrm{V}}h_3.
\end{aligned} \tag{62}$$

These relations are modified when the MC hypothesis is relaxed. In particular the constraint (III), as happens in the ordinary sector at order p^4, is no more required by the equivalence of the two models.

We analyse in more detail the equivalence of these models. We proved that they generate the same set of divergent operators (i.e. with divergent counterterms in the one-loop effective lagrangian). Comparing the solutions of the equations of motion (40) and (57) we find that the field strength $\rho_{\mu\nu}$ in the HGS model plays the same role as the tensor field $V_{\mu\nu}$ in the T model. The vector building blocks in the two cases are $\rho_{\mu\nu}$ and $V_{\mu\nu}$ and their covariant derivatives and we conclude that the two models are equivalent in a strong sense, i.e. they generate the same set of divergent and also finite terms in the effective theory. Moreover, the solution at the lowest order is in both cases proportional to the "ordinary" current $J^\mathrm{O}(p^2)$.

Provided the relation $F_V = 2G_V$ is valid in the T model, this in turn can be written in both cases as $J^{O}_{\mu\nu}(p^2) = C\Upsilon_{\mu\nu}$, i.e. proportional to the current $\Upsilon_{\mu\nu}$ defined in (7). As Akhoury found, the $\Upsilon_{\mu\nu}$ operator is one of the building blocks of the $\Delta L^{\infty}_{1\,\mathrm{loop}}$. In this way there is a direct correspondence between terms with two Υ (one Υ) in the $\Delta L^{\infty}_{1\,\mathrm{loop}}$ and terms with two vectors (one vector) in the HGS and Tensor models as implied by the MC hypothesis.

All the terms in photon-pseudoscalar sector of the effective lagrangian are produced with the same relative weights as in the divergent one-loop lagrangian $\Delta L^{\infty}_{1\,\mathrm{loop}}$. This suggests that they may play a role not only in the saturation of the finite part of the counterterms but also in the cancellation of the divergent part. This speculation has to be further investigated *.

The strong equivalence between HGS and T models implies that the discussion of the equivalence with the CV model is the same as the one in subsect. 4.2.

5. Phenomenological implications and phenomenological constraints

We have established the strong equivalence of the Tensor and HGS models and the weak equivalence of both of them with the Conventional Vector model. The latter will not be further considered in the discussion of the phenomenological constraints. The analysis will be performed for the first two models in order to determine the values of the couplings and to verify that they do not contradict the relations (52) and (62) imposed by the equivalence.

There are constraints on some of the coefficients present in the anomalous sector coming from the analysis of the lowest order ordinary sector which we are going to summarize.

Predictions for this set of coefficients come from measurements of ordinary processes involving vector mesons, pseudoscalars and external gauge bosons: $\rho \to \pi\pi$, $\rho \to e^+e^-$ and $\omega \to e^+e^-$. There are two relevant couplings in each model. They are g and f in the HGS model (g is the gauge coupling of the vector meson and f is the pion decay constant), F_V and G_V in the Tensor model. A complete analysis of the ordinary sector at $O(p^4)$ was done in ref. [2] for the Tensor model. The couplings F_V and G_V can be fixed through the processes $\rho \to e^+e^-$ and $\rho \to \pi\pi$ respectively. Using the most recent experimental data [11] $\Gamma(\rho^0 \to e^+e^-) = (6.77 \pm 0.32)$ keV and $\Gamma(\rho \to \pi\pi) = (149 \pm 3)$ MeV, we obtain $|F_V| = 153$ MeV and $|G_V| = 68$ MeV.

Good values for g and f are $g = 4.1$ and $f = 132$ MeV, which satisfy the relation $\sqrt{2}gf = m_\rho \simeq 765$ MeV, close to the experimental value $m_\rho = 768.3$ MeV. The only datum that we cannot fully explain is the $\rho \to e^+e^-$ decay, which comes out a little bit low. The old problem of the discrepancy between the experimental

* We thank J. Bijnens, G. Ecker and J. Gasser for a clarifying discussion on this point.

ratio of the widths $\rho \to e^+e^-$ (Γ_ρ^l) and $\omega \to e^+e^-$ (Γ_ω^l) and the prediction from the nonet symmetry hypothesis ($(\Gamma_\rho^l)/(\Gamma_\omega^l) = 9$) remains open in this context.

A detailed analysis in ref. [5] of the equivalence of the two models in the ordinary sector at $O(p^4)$ of the effective theory has shown that the relation $F_V = 2G_V$ is required for their equivalence. The authors derive the additional approximate equation $F_V G_V = F_\pi^2$ ($F_\pi = 93$ MeV) using an unsubtracted dispersion relation for the pion form factor. With these two relations they find $F_V = 132$ MeV and $G_V = 66$ MeV, in good agreement with the experiment.

A set of constraints on the values of the couplings in the anomalous sector of the resonance models comes from the strong and electromagnetic decays of resonances like $\omega \to 3\pi$, $V \to \pi\gamma$ and $V \to \pi l^+ l^-$ (i.e. with the production of a lepton pair). All previous works on the anomalous sector formulate the resonance model in the HGS scheme. There are two main differences with our approach in the same model: (i) some of the possible invariants (also in the divergent set) were not considered before, (ii) the basis of counterterms was not restricted by the MC hypothesis. Following refs. [9,10] three couplings a_1, a_2 and a_3 appear in the lagrangian, while in our scheme only the term with coupling a_2 (in our notation h_1) appears and $a_1 = a_3 = 0$. This reduction makes the comparison with experimental data very severe. Surprisingly, the MC scheme works reasonably well and can be used to improve the predictivity for low energy processes.

To show the phenomenological implications of the MC scheme at $O(p^6)$ we analyse some resonance processes first in the HGS model, comparing our results also with the non-minimal predictions, and then in the Tensor model.

A reasonable choice is to find a range of values for each coupling such that we have agreement with all the experimental data within 10–15%. The range of g, from the $\rho \to \pi\pi$, $\rho \to e^+e^-$ and $\omega \to e^+e^-$ decays, is 4.0–4.2. From the radiative decay $\omega \to \pi^0\gamma$, with the minimal choice, the parameter h_1 is a function of g and the coupling to photon g_ω, which is experimentally determined:

$$h_1 = \frac{g_\omega^{\exp}}{2eg}.$$

This gives a range for h_1 equal to $-0.034 \div -0.039$ for $g = 4.2$ and $-0.036 \div -0.041$ for $g = 4.0$. These values of h_1 correspond to the old relation $a_2 + 2a_3 = -3/8\pi^2 \simeq -0.038$ used in the non-minimal choice. The range of values found for h_1 are in very good agreement with the width of the $\omega \to 3\pi$. In the non-minimal model the fit with experimental data for this process leads to a constraint between parameters a_1 and a_2 ($a_2 \equiv h_1$) which is represented by the ellipse in fig. 1. The MC solution $h_1 = -0.036 \div -0.034$ and $a_1 = 0$ lies satisfactorily on the ellipse, given the sensitivity of the $\omega \to 3\pi$ width to the pion decay constant f and to the coupling g (g^2/f^6).

The last process that we consider is the $e^+e^- \to \omega, \rho \to \pi^0\mu^+\mu^-$ at the omega

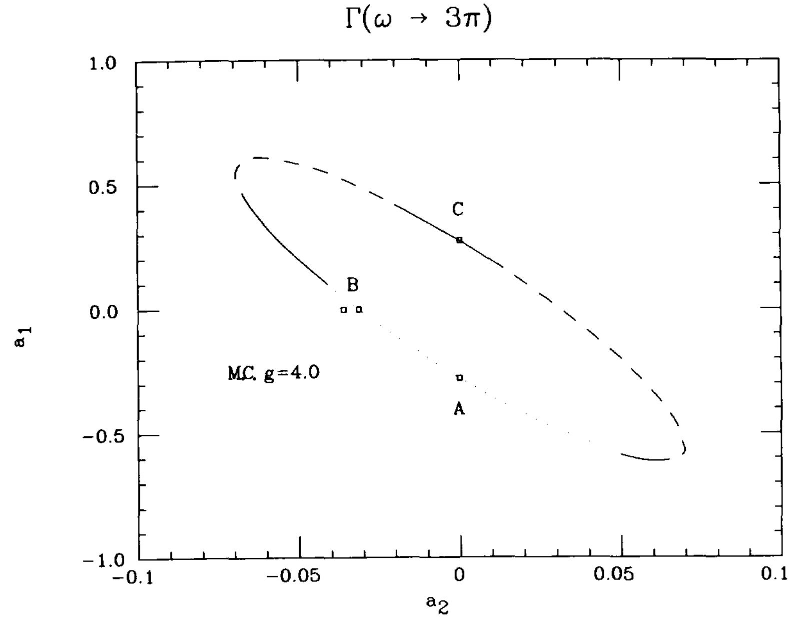

Fig. 1. The ellipse defines the values of parameters a_1 and a_2 [10] which give the correct width for the $\omega \to 3\pi$. Dotted, solid and dashed portions of the ellipse denote values of the parameters a_1 and a_2 whose corresponding cross sections for the process $e^+e^- \to 3\pi$ are within one, two or more standard deviations from the present experimental data respectively (see ref. [10] for further details). Within the MC scheme we reduce the ellipse to the point $a_1 = 0$, $h_1 \equiv a_2 = -0.036$, with $g = 4.0$.

peak ($s = m_\omega^2$), which was studied in the non-minimal framework in ref. [12]. In this case we obtain again a good agreement with the experimental value of 0.164 ± 0.040 nb on the omega peak, with a prediction ranging between 0.18 and 0.24 nb. Another interesting feature of this process is the dependence upon the muon pair invariant mass k^{*2} (see ref. [12] for details), which can be parameterized in terms of the usual e.m. form factor $F = \Lambda^2/(\Lambda^2 - k^{*2})$. In our scheme it is equal to $1 + 2k^{*2}/m_\rho^2$, which corresponds to a value for Λ given by $\Lambda \simeq m_\rho/\sqrt{2} \simeq$ 0.55 GeV, to be compared with the experimental value of $\Lambda_{\text{exp}} = 0.65 \pm 0.03$ GeV.

We now consider the same two processes $\omega \to \pi^0\gamma^*$ and $\omega \to 3\pi$ in the Tensor model.

The diagrams that mediate the two processes in the T and HGS models are shown in fig. 2. In the Tensor model only the vector–vector–pseudoscalar vertex VVP is present in the anomalous sector, while in the HGS model also the direct vector–three pseudoscalars VPPP and vector–pseudoscalar–photon VPγ vertices are present. In eq. (63) we give the list of the interaction terms (anomalous and ordinary terms) that enter in the two models. They all come from term (1) in the divergent set of the two models (see the lists (25) and (37)) by expanding the U

field in powers of the pseudoscalar matrix Φ and introducing the diagonal quark charge matrix Q:

Tensor

$$\mathrm{VVP} = -\frac{2t_1}{f}\epsilon^{\mu\nu\alpha\beta}\left\langle V_{\mu\nu}\left\{\partial^\lambda V_{\alpha\lambda}, \partial_\beta \Phi\right\}\right\rangle,$$

$$\mathrm{VPP} = \frac{4iG_V}{\sqrt{2}f^2}\left\langle V_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi\right\rangle,$$

$$\mathrm{V}\gamma = \frac{eF_V}{\sqrt{2}}F^{\mu\nu}\langle V_{\mu\nu}Q\rangle.$$

HGS

$$\mathrm{VVP} = -\frac{4g^2}{f}h_1\epsilon^{\mu\nu\alpha\beta}\langle\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Phi\rangle,$$

$$\mathrm{VPPP} = -\frac{4ig}{f^3}h_1\epsilon^{\mu\nu\alpha\beta}\langle\rho_\mu\partial_\nu\Phi\partial_\alpha\Phi\partial_\beta\Phi\rangle,$$

$$\mathrm{VP}\gamma = \frac{2eg}{f}h_1\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu\langle Q\{\partial_\alpha\rho_\beta, \Phi\}\rangle,$$

$$\mathrm{VPP} = 2ig\left\langle\rho_\mu\partial^\mu\Phi\Phi\right\rangle,$$

$$\mathrm{V}\gamma = -2egf^2A^\mu\langle\rho_\mu Q\rangle. \tag{63}$$

The vector propagators in the two models are

$$P^{\mathrm{T}}_{\mu\nu,\rho\sigma} = \frac{i}{M^2(M^2-k^2-i\epsilon)}\left[g_{\mu\rho}g_{\nu\sigma}(M^2-k^2) + g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - (\mu\leftrightarrow\nu)\right],$$

$$P^{\mathrm{HGS}}_{\mu\nu} = \frac{i}{M^2-k^2-i\epsilon}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right). \tag{64}$$

The Tensor propagator contains a contact term $\delta^4(x)$ not present in the usual

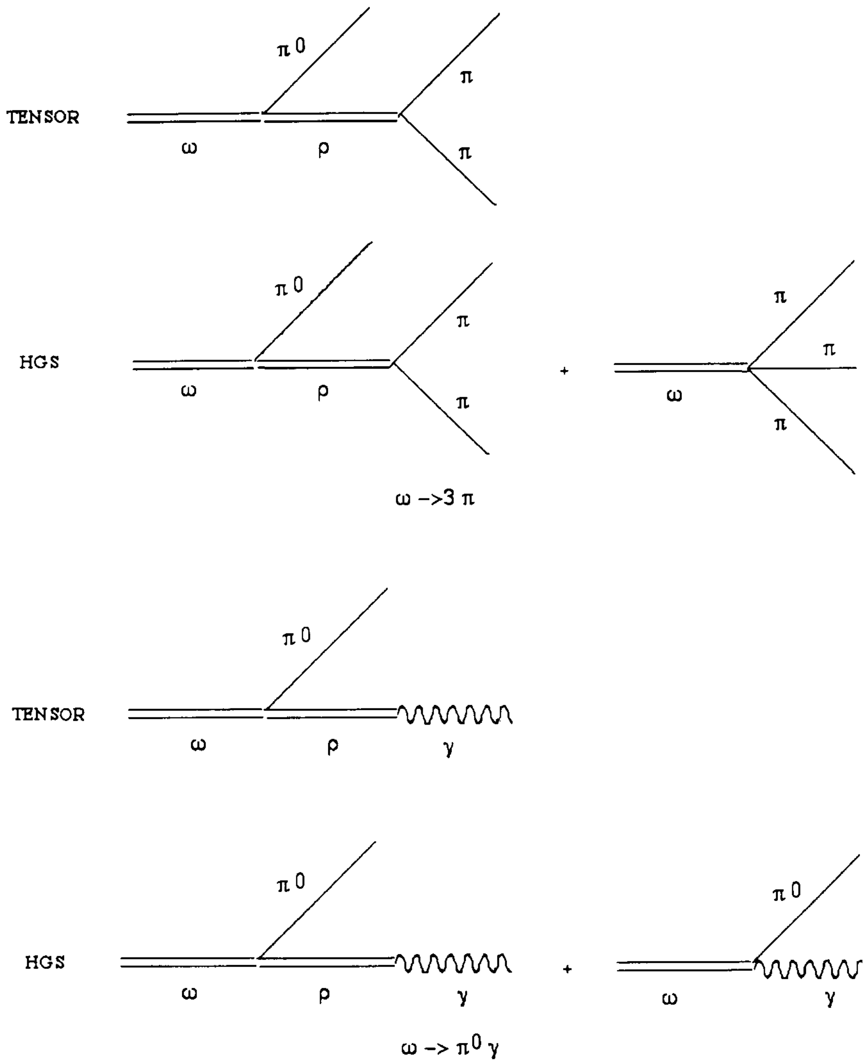

Fig. 2. Lowest order diagrams with vector mesons exchange for the processes $\omega \to \pi^0\gamma$ and $\omega \to 3\pi$ within the HGS and T models.

propagator of a massive vector field and the corresponding normalization of the tensor field is

$$\langle 0 | V_{\mu\nu} | V, p \rangle = \frac{i}{M}(p_\mu \epsilon_\nu - p_\nu \epsilon_\mu). \tag{65}$$

The amplitude calculated in the two models for the process $\omega \to \pi^0 \gamma^*$ is

$$A^{\mathrm{T}}(\omega \to \pi^0(p)\gamma^*(k)) = \frac{8eF_V t_1}{\sqrt{2}\, fM} \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}(k) k^{\nu} \epsilon_{\omega}^{\alpha}(k_{\omega}) p^{\beta} \left[1 + 2\frac{k^2}{M^2 - k^2}\right],$$

$$A^{\mathrm{HGS}}(\omega \to \pi^0(p)\gamma^*(k)) = \frac{2eg}{f} h_1 \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}(k) k^{\nu} \epsilon_{\omega}^{\alpha}(k_{\omega}) p^{\beta} \left[1 + 2\frac{k^2}{M^2 - k^2}\right], \tag{66}$$

where M is the average vector resonances mass. In order to get the same prediction in the two models the couplings must be related as follows:

$$4F_V t_1 = \sqrt{2}\, gMh_1. \tag{67}$$

The equivalence of the T and the HGS models at the effective level and in particular eqs. (I) and (II) in (62) imply the relation above.

Another constraint on G_V comes from the $\omega \to 3\pi$ process. The amplitude in the two models is

$$A^{\mathrm{T}}(\omega \to 3\pi) = -i\frac{96 G_V t_1}{\sqrt{2}\, f^3 M} \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu} p_+^{\nu} p_-^{\alpha} p_0^{\beta} \times \left[1 - \tfrac{2}{3}\Sigma_{(ij)} \frac{M^2}{M^2 - p_{ij}^2 - iM\Gamma_{\rho}}\right],$$

$$A^{\mathrm{HGS}}(\omega \to 3\pi) = -i\frac{12g}{f^3} h_1 \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu} p_+^{\nu} p_-^{a} p_0^{\beta} \times \left[1 - \tfrac{2}{3}\Sigma_{(ij)} \frac{M^2}{M^2 - p_{ij}^2 - iM\Gamma_{\rho}}\right]. \tag{68}$$

The corresponding relation for G_V is

$$8G_V t_1 = \sqrt{2}\, gMh_1, \tag{69}$$

which is valid because of eqs. (I), (II) and (III) in (62).

In table 2 we summarize the experimental data on the anomalous processes that we have considered above and the relevant combinations of couplings present in the amplitude of each process in turn, both in the HGS and T models within the MC scheme. The theoretical predictions have been obtained with the parameter h_1 fixed to -0.036 and $g = 4.0$.

TABLE 2

The combinations of couplings in the second and third columns for the two models multiply the same amplitude for the corresponding process in the first column. The two sets are totally equivalent through equations in (62). The last column represents the theoretical fit which is therefore identical in the two cases and, once the constraints in the non-anomalous sector are fulfilled, is obtained with a single parameter. The values in the table are obtained with $h_1 = -0.036$ and $g = 4.0$.

Anomalous process	HGS	T	exp [11]	th
$\Gamma(\omega \to \pi^0\gamma)$	$\frac{2eg}{f}h_1$	$\frac{8eF_V t_1}{\sqrt{2}fM}$	0.72 ± 0.05 MeV	0.63
$\Gamma(\omega \to 3\pi)$	$-\frac{4g}{f^3}h_1$	$-\frac{32G_V t_1}{\sqrt{2}f^3M}$	7.49 ± 0.10 MeV	8.1
$\sigma(e^+e^- \to \omega, \rho \to \pi^0\mu^+\mu^-)_{s=m_\omega^2}$	$e^4g^2fh_1$	$\frac{2e^4F_V^2t_1}{f}$	0.164 ± 0.040 nb	0.18

The equivalence between the two models can be shown to work for the whole s dependence of the process $e^+e^- \to 3\pi$: the prediction in the MC scheme corresponds to the curve B of ref. [10]. The equivalence between the two models in all the processes can be traced back to the fact that a single amplitude for on-shell hadrons in the tensor model is replaced in the HGS model by an equivalent sum of amplitudes:

$$(\text{VVP} \times \text{V}\gamma)^{\text{T}} = (\text{VP}\gamma + \text{VVP} \times \text{V}\gamma)^{\text{HGS}},$$

$$(\text{VVP} \times \text{VPP})^{\text{T}} = (\text{VPPP} + \text{VVP} \times \text{VPP})^{\text{HGS}}. \tag{70}$$

This property suggests the possibility of a complete equivalence of the two models for on-shell amplitudes already at the resonance lagrangian level that we are currently investigating.

The reduction to a single coupling in the anomalous sector with pseudoscalars and photons allows us to determine its value independently from any vector model. It can be obtained from the dependence of the $\eta \to \mu^+\mu^-\gamma$ decay upon the muon pair invariant mass [6]. The experimental value of 1.9 ± 0.4 GeV^{-2} translates into a value for $h_1 \simeq -0.037 \pm 0.001$, which agrees with the value used in table 2.

6. Conclusions

The equivalence among the three vector models that we have analysed (CV, T and HGS), as far as the effective anomalous lagrangian at order p^6 is concerned, holds in a strong sense between the last two, while the first model cannot generate all the terms of the divergent one-loop lagrangian of the other two models nor any finite term. These extra terms (finite or divergent) as well as those which cannot be obtained by any vector model can be produced by the exchange of scalars, pseudoscalars, axials and singlets analogously to the non-anomalous case. The number of independent terms of the effective lagrangian can be reduced to a minimal set by defining the "Minimal Coupling" scheme. This can be naturally implemented in the HGS and T models by introducing two-vector anomalous interaction terms and correspondingly restricting the one-vector sector.

The coefficients of the operators generated in the photon–pseudoscalar sector of the effective lagrangian by resonance models appear with the same relative weights as in the divergent one-loop anomalous pseudoscalar lagrangian. This suggests a possible role of the resonance parameters, not only in the saturation of the finite part of the counterterms, but also in the cancellation of their divergent part.

The couplings in the lagrangian have to be fixed from resonance decay processes. In spite of the couplings reduction of the MC scheme the agreement with experimental data remains satisfactory for both HGS and T models, with the correspondence among the parameters of the two models established from the effective lagrangian equivalence requirement. This may hide a higher degree of equivalence, valid for all on-shell amplitudes in the two models which would become, if this was the case, a simple reparameterization one of the other.

The photon–pseudoscalar sector of the one-loop anomalous lagrangian receives contributions from vectors only. The saturation of the finite part of the counterterms can be discussed by considering just vector resonances.

In a model not restricted by the MC hypothesis the check of the saturation would need the determination of the finite values of all the independent counterterms from low energy photon-pseudoscalar processes and the comparison with the predictions of vector exchange.

In the MC scheme the latter are parameterized in terms of a single parameter. If one assumes that also among the counterterms the same relations implied by the MC scheme hold, one is left with a corresponding single counterterm to be fixed from low energy data. The $\eta \to \gamma\mu^+\mu^-$ decay allows us to determine such a counterterm and to compare it successfully with the vector exchange prediction. Given the assumption above, the saturation hypothesis is confirmed in the photon-pseudoscalar sector.

We thank J. Gasser for stimulating discussions about the equivalence of different vector models in the ordinary sector and A. Bramon, A. Grau and G. Pancheri for making their results on the process $e^+e^- \to \pi^0\mu^+\mu^-$ available to us before publication.

Appendix A

IDENTITIES FOR THE EXTERNAL CURRENTS OPERATORS

We give some useful relations valid for the octet operators u_μ, $f_{\mu\nu}^{\pm}$, $\chi_{\pm}$ etc., that we use to construct the external currents in the vector resonance models.

Let us define $\mathscr{L}_\mu \equiv U^\dagger D_\mu U$ and the building blocks which transform as octets under $SU(3)_L \times SU(3)_R$

$$\begin{aligned}
u_\mu &= iu^\dagger D_\mu D u^\dagger = u_\mu^\dagger = iu\mathscr{L}_\mu u^\dagger, \\
u_{\mu\nu} &= iu^\dagger D_\mu D_\nu U u^\dagger, \\
f_{\mu\nu}^{\pm} &= uL_{\mu\nu}u^\dagger \pm u^\dagger R_{\mu\nu} u \equiv u\left(L_{\mu\nu} \pm \tilde{R}_{\mu\nu}\right)u^\dagger, \\
\chi_{\pm} &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u.
\end{aligned} \tag{A.1}$$

For the covariant derivative of an octet operator Θ the following relation holds

$$\nabla_\mu \Theta = u^\dagger D_\mu(u\Theta u)u^\dagger + \frac{i}{2}\{u_\mu, \Theta\}, \tag{A.2}$$

from which follow some useful identities:

$$\begin{aligned}
\nabla_\mu u_\nu &= iuD_\mu \mathscr{L}_\nu u^\dagger + \frac{i}{2}u\left[\mathscr{L}_\mu, \mathscr{L}_\nu\right]u^\dagger, \\
\nabla_\mu f_{\alpha\beta}^{+} &= u\left(D_\mu L_{\alpha\beta} + U^\dagger D_\mu R_{\alpha\beta} U + \tfrac{1}{2}\left[\mathscr{L}_\mu, L_{\alpha\beta} - \tilde{R}_{\alpha\beta}\right]\right)u^\dagger.
\end{aligned} \tag{A.3}$$

Appendix B

RULES FOR CONSTRUCTING THE INTERACTION VECTOR LAGRANGIAN

All the possible terms which mediate the interaction of vector mesons with the external currents have to be chiral invariant terms, Lorentz invariant and P and C conserving. To reduce the most general set of terms, that are in general not all independent, to the subset of independent terms (i.e. a basis) we have used three tools:

(i) The existence of relations among some of the building blocks of the external currents.

(ii) The equation of motion of the pseudoscalar field.

(iii) The Schouten identity.

(i) Two relations reduce the number of independent $O(p^2)$ octet operators:

$$\nabla_\mu u_\nu = u_{\mu\nu} + \frac{i}{2}\{u_\mu, u_\nu\},$$

$$\nabla_{[\mu} u_{\nu]} = -f^-_{\mu\nu}. \tag{B.1}$$

(ii) The equation of motion at order p^2 (we need the solution up to this order) for the pseudoscalar field is

$$\nabla^\lambda u_\lambda = -\frac{i}{2}\chi_- + \frac{i}{3}\langle\chi_-\rangle. \tag{B.2}$$

With our basis we always choose $\nabla^\lambda u_\lambda$ (or χ_-) and the singlet $\langle\chi_-\rangle$ as independent operators.

(iii) The Schouten identity originates from the property that a five indices tensor, completely antisymmetric, is null in four dimensions. This implies the identity

$$\delta_{\rho\sigma}\epsilon_{\mu\nu\alpha\beta} - \delta_{\rho\mu}\epsilon_{\sigma\nu\alpha\beta} - \delta_{\rho\nu}\epsilon_{\mu\sigma\alpha\beta} - \delta_{\rho\alpha}\epsilon_{\mu\nu\sigma\beta} - \delta_{\rho\beta}\epsilon_{\mu\nu\alpha\sigma} = 0. \tag{B.3}$$

A typical anomalous interaction term with vector mesons is a trace over the product of operators with the following general form:

$$\epsilon^{\mu\nu\alpha\beta}\left\langle \Theta_{1\lambda}\Theta^\lambda_{2\mu\nu\alpha\beta}\right\rangle \to \delta_{\rho\sigma}\epsilon^{\mu\nu\alpha\beta}\left\langle \Theta^\rho_1\Theta^\sigma_{2\mu\nu\alpha\beta}\right\rangle.$$

The identity (B.3) then relates terms which differ by a reordering of the Lorentz indices. We give an example. Let us consider three two-vector terms in the Tensor model

$$(1) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle V_{\mu\nu}\left\{\nabla^\lambda V_{\alpha\lambda}, u_\beta\right\}\right\rangle,$$

$$(2) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle V_{\mu\lambda}\left\{\nabla_\nu V_{\alpha\lambda}, u_\beta\right\}\right\rangle,$$

$$(3) \quad \epsilon^{\mu\nu\alpha\beta}\left\langle V_{\mu\nu}\left\{\nabla_\alpha V_{\beta\lambda}, u^\lambda\right\}\right\rangle. \tag{B.4}$$

Using the identity (B.3) we easily find a dependence relation among the three terms:

$$[(1) + (3)] - [(2) \times (2)] = 0. \tag{B.5}$$

Analogous relations permit us to reduce a number of dependent terms among all the possible invariants one can construct with only one vector meson.

Appendix C

EFFECTIVE CONTRIBUTIONS FROM AXIAL, SCALAR, PSEUDOSCALAR AND FLAVOUR SINGLET RESONANCE EXCHANGE

In the construction of the interaction lagrangian for an axial resonance field in the CV model one encounters the problem that the chiral symmetry allows for mixing terms with the pseudoscalar field, like the term $\langle A_\mu u^\mu \rangle$ which already occurs at $O(p)$. A shift of the field

$$A'_\mu = A_\mu + c u_\mu + \dots \tag{C.1}$$

removes the spin-0 component of the axial field. We refer to the shifted field from now on. The interaction lagrangian among one axial vector and the lowest order "ordinary" and "anomalous" external currents is

$$L_\mathrm{I} = \left\langle A_\mu J^\mu(p^3) \right\rangle + \left\langle A_{\mu\nu} J^{\mu\nu}(p^2) \right\rangle, \tag{C.2}$$

where $A_{\mu\nu}$ is the usual antisymmetrized covariant derivative $A_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ and the currents are (for simplicity we drop the couplings in front of each term)

$$\begin{aligned} J_\mu &= \left[u^\lambda, f^+_{\mu\lambda}\right] + u_\lambda u_\mu u^\lambda + \left\{u_\mu, u_\lambda u^\lambda\right\} \\ &\quad + \epsilon_{\mu\nu\alpha\beta}\left\{u^\nu, f_-^{\alpha\beta}\right\} + \left\{u_\mu, \chi_+\right\} + u_\mu \left\langle u_\lambda u^\lambda \right\rangle \\ &\quad + u_\mu \langle \chi_+ \rangle + u_\lambda \langle u^\lambda u_\mu \rangle, \\ J_{\mu\nu} &= f^-_{\mu\nu}. \end{aligned} \tag{C.3}$$

The solution of the equation of motion leads to the effective lagrangian

$$L_\mathrm{A}^\mathrm{eff}(p^6) = -\frac{1}{2M_\mathrm{A}^2}\langle J_\mu J^\mu \rangle - \frac{2}{M_\mathrm{A}^2}\langle \nabla_\mu J_\nu J^{\mu\nu} \rangle + O(p^8), \tag{C.4}$$

which contributes to the effective $O(p^6)$ lagrangian in the anomalous sector:

$$\begin{aligned} L_\mathrm{A}^\mathrm{odd} = -\frac{1}{2M_\mathrm{A}^2}\epsilon_{\mu\nu\alpha\beta}\Big\{ &\tfrac{1}{2}\left\langle [u^\mu, u^\nu][f_-^{\alpha\beta}, \chi_+] \right\rangle \\ &+ \tfrac{1}{2}\left\langle [u^\mu, u^\nu][f_-^{\alpha\beta}, u_\lambda u^\lambda] \right\rangle + \left\langle \{u^\nu, f_-^{\alpha\beta}\} u_\lambda \right\rangle \left\langle u^\lambda u_\mu \right\rangle \\ &+ \text{finite contributions}\Big\}. \end{aligned} \tag{C.5}$$

If axials are introduced in a tensor form they all give contributions proportional to $f^-_{\mu\nu}$ and they do not contaminate the photon–pseudoscalar sector.

For a scalar resonance $S(0^{++})$ the interaction lagrangian with external currents up to order p^4 is

$$L_{\mathrm{I}} = \langle SJ_{\mathrm{S}}^{\mathrm{O}}(p^2)\rangle + \langle SJ_{\mathrm{S}}^{\mathrm{A}}(p^4)\rangle, \tag{C.6}$$

where the currents are

$$\begin{aligned} J_{\mathrm{S}}^{\mathrm{O}}(p^2) &= u_\lambda u^\lambda + \chi_+, \\ J_{\mathrm{S}}^{\mathrm{A}}(p^4) &= \epsilon_{\mu\nu\alpha\beta}\left[f_+^{\mu\nu}, f_-^{\alpha\beta}\right] + \epsilon_{\mu\nu\alpha\beta}\left[u^\mu u^\nu, f_-^{\alpha\beta}\right] + \epsilon_{\mu\nu\alpha\beta}u^\mu\langle u^\nu f_-^{\alpha\beta}\rangle. \end{aligned} \tag{C.7}$$

Inserting the solution of the equation of motion, the effective $O(p^6)$ lagrangian in the anomalous sector reads

$$\begin{aligned} L_{\mathrm{S}}^{\mathrm{odd}} = \frac{1}{M_{\mathrm{S}}^2}\epsilon_{\mu\nu\alpha\beta}\Big\{&\langle\left[f_+^{\mu\nu}, f_-^{\alpha\beta}\right]u_\lambda u^\lambda\rangle \\ &+\langle\left[f_+^{\mu\nu}, f_-^{\alpha\beta}\right]\chi_+\rangle \\ &+\langle\left[u^\mu u^\nu, f_-^{\alpha\beta}\right]u_\lambda u^\lambda\rangle + \langle\left[u^\mu u^\nu, f_-^{\alpha\beta}\right]\chi_+\rangle \\ &+\langle u_\lambda u^\lambda u^\mu\rangle\langle u^\nu f_-^{\alpha\beta}\rangle + \langle\chi_+ u^\mu\rangle\langle u^\nu f_-^{\alpha\beta}\rangle\Big\}. \end{aligned} \tag{C.8}$$

In the case of a pseudoscalar resonance $P(0^{-+})$ we have

$$L_{\mathrm{I}} = \langle PJ_{\mathrm{P}}^{\mathrm{O}}(p^2)\rangle + \langle PJ_{\mathrm{P}}^{\mathrm{A}}(p^4)\rangle, \tag{C.9}$$

where the currents are

$$\begin{aligned} J_{\mathrm{P}}^{\mathrm{O}}(p^2) &= \chi_-, \\ J_{\mathrm{P}}^{\mathrm{A}}(p^4) &= \epsilon_{\mu\nu\alpha\beta}\left\{f_-^{\mu\nu}f_-^{\alpha\beta} + f_+^{\mu\nu}f_+^{\alpha\beta} + \left\{u^\mu u^\nu, f_+^{\alpha\beta}\right\}\right\}. \end{aligned} \tag{C.10}$$

The effective $O(p^6)$ lagrangian in the anomalous sector reads

$$\begin{aligned} L_{\mathrm{P}}^{\mathrm{odd}} = \frac{1}{M_{\mathrm{P}}^2}\epsilon_{\mu\nu\alpha\beta}\Big\{&\langle\chi_- f_-^{\mu\nu}f_-^{\alpha\beta}\rangle + \langle\chi_- f_+^{\mu\nu}f_+^{\alpha\beta}\rangle \\ &+\langle\chi_-\left\{u^\mu u^\nu, f_+^{\alpha\beta}\right\}\rangle\Big\}. \end{aligned} \tag{C.11}$$

The interaction lagrangian for a flavour scalar singlet S_1 at lowest order in the non-anomalous and anomalous sector is given by

$$L_{\mathrm{I}} = S_1 J_{\mathrm{S}_1}^{\mathrm{O}}(p^2) + S_1 J_{\mathrm{S}_1}^{\mathrm{A}}(p^4), \tag{C.12}$$

with the currents

$$J_{S_1}^{O}(p^2) = \langle u_\lambda u^\lambda \rangle + \langle \chi_+ \rangle,$$

$$J_{S_1}^{A}(p^4) = J_{WZ}. \tag{C.13}$$

Given the C and P transformation properties of the scalar singlet S_1 the anomalous current of order p^4 is just the Wess–Zumino term J_{WZ}. In the case of a flavour pseudoscalar singlet P_1 the interaction lagrangian is

$$L_O = P_1 J_{P_1}^{O}(p^2) + P_1 J_{P_1}^{A}(p^4). \tag{C.14}$$

Both terms are proportional to the ordinary current in the effective lagrangian which reads:

$$J_{P_1}^{O}(p^2) = \langle \chi_- \rangle \tag{C.15}$$

and they contribute only to finite terms vanishing in the exact SU(2) isospin limit.

Also flavour scalar singlets produce some of the finite terms in the effective anomalous lagrangian.

The scalar resonances produce all the terms needed to restore the equivalence of the CV model with the T and HGS models in the anomalous sector, the axial ones produce some of them and the pseudoscalars none of them.

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